# Blind Estimation of Carrier Frequency Offset for OFDM Systems using Maximum Likelihood Technique

**A Dissertation submitted**
**in partial fulfillment of the requirements for the award of the Degree of**

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**in**

**Electrical Engineering**
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**Submitted By**


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# CERTIFICATE

*This is to certify that the dissertation entitled* ***"Blind Estimation of Carrier Frequency Offset for OFDM Systems using Maximum Likelihood Technique"*** *submitted by* ***Mr. Raj Kumar Ranjan*** *(Roll No.: 211EE1327), to the Department of Electrical Engineering, National Institute of Technology Rourkela, in partial fulfillment of the requirements for the award of the degree "****MASTER OF TECHNOLOGY" in Electrical Engineering (Electronic Systems & Communication )*** *is an authentic work carried out at Department of Electrical Engineering, National Institute of Technology, Rourkela by him under my supervision and guidance.*

*To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any degree or diploma.*

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## CANDIDATE'S DECLARATION

*I hereby declare that the work presented in this dissertation entitled* ***"Blind Estimation of Carrier Frequency Offset for OFDM Systems using Maximum Likelihood Technique",*** *is an authentic record of my own work carried out at Department of Electrical Engineering, National Institute of Technology, Rourkela as requirements for the award of degree of Master of Technology (M.Tech.) in Electrical Engineering (Electronic Systems & Communication Engineering), submitted in the National Institute of Technology, Rourkela for the session from June 2011 to June 2014 under supervision of* ***Prof. P.K. Sahu****, Department of Electrical Engineering, National Institute of Technology, Rourkela.*

*The matter embodied in the thesis has not been submitted to any other University/Institute for the award of any degree or diploma.*

RAJ KUMAR RANJAN
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***Dedicated***

***To***

***My Beloved***

***Family***

*Whose blessings, love and sacrifice brought me here up to ……….*

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# ABSTRACT

A Multicarrier Communication system such as an Orthogonal Frequency Division Multiplexing OFDM has been shown to be an impressive approach to combat multipath fading in wireless communications. OFDM is a modulation scheme that allows digital data to be efficiently and reliably transmitted over a radio channel, even in multipath environments. OFDM transmits data by using a large number of narrow bandwidth carriers. These carriers are regularly spaced in frequency, forming a block of spectrum. The frequency spacing and time synchronization of the carriers is chosen in such a way that the carriers are orthogonal, meaning that they do not cause interference to each other. In spite of the success and effectiveness of the OFDM systems, it suffers from a well-known drawback of high sensitivity to Carrier Frequency Offset (CFO). The presence of the CFO in the received carrier will lose orthogonality among the carriers and causes a reduction of desired signal amplitude in the output decision variable and introduces Inter Carrier Interference (ICI). It then brings up an increase of Bit Error Rate (BER). This makes the problem of estimating the CFO an attractive and necessary research problem. In this thesis Blind Modified ML CFO estimation technique based on data symbol repetition is discussed to estimate the offset parameter.

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# ABBREVIATIONS

| | | |
|---|---|---|
| 1 | OFDM | Orthogonal Frequency Division Multiplexing |
| 2 | OFDMA | Orthogonal Frequency Division Multiple Access |
| 3 | CFO | Carrier Frequency Offset |
| 4 | ICI | Inter Carrier Interference |
| 5 | ML | Maximum Likelihood |
| 6 | GSM | Global System for Mobile |
| 7 | ISI | Inter Symbol Interference |
| 8 | LOS | Line of Sight |
| 9 | NLOS | Non Line of Sight |
| 10 | DAB | Digital Audio Broadcasting |
| 11 | DVB | Digital Video Broadcasting |
| 12 | HDTV | High-Definition Television |
| 13 | DSL | Digital Subscriber Line |
| 14 | ADSL | Asymmetric Digital Subscriber Line |
| 15 | VDSL | Very High Data Rate Digital Subscriber Line |
| 16 | LAN | Local Area Network |
| 17 | MIMO | Multiple Input Multiple Output |
| 18 | PAPR | Peak-to-Average Power Ratio |
| 19 | SC-FDMA | Single Carrier Frequency Division Multiple Access |
| 20 | STBC | Space-Time Block Coding |
| 21 | SNR | Signal-to-Noise Ratio |
| 22 | LTE | Long Term Evolution |
| 23 | BICM | Bit Interleaved Coded Modulation |
| 24 | TDMA | Time Division Multiple Access |
| 25 | FDMA | Frequency Division Multiple Access |
| 26 | CDMA | Code Division Multiple Access |
| 27 | FDM | Frequency Division Multiplexing |
| 28 | FDD | Frequency Division Duplexing |

| | | |
|---|---|---|
| 29 | TDD | Time Division Duplexing |
| 30 | MCM | Multi Carrier Modulation |
| 31 | AMPS | Advanced Mobile Phone Service |
| 32 | GMSK | Gaussian Minimum Shift Keying |
| 33 | DMT | Discrete Multi-Tone |
| 34 | DFT | Discrete Fourier Transform |
| 35 | IDFT | Inverse Discrete Fourier Transform |
| 36 | FFT | Fast Fourier Transform |
| 37 | CP | Cyclic Prefix |
| 38 | GI | Guard Interval |
| 39 | VLSI | Very Large Scale Integration |
| 40 | SHF | Super High Frequency Band |
| 41 | EHF | Extremely High Frequency Band |
| 42 | BPSK | Binary Phase Shift Keying |
| 43 | QPSK | Quadrature Phase Shift Keying |
| 44 | QAM | Quadrature Amplitude Modulation |
| 45 | VC | Virtual Carriers |
| 46 | AWGN | Additive White Gaussian Noise |
| 47 | CIR | Carrier to Interference Ratio |
| 48 | MLE | Maximum Likelihood Estimate |
| 49 | CM | Constant Modulus |
| 50 | PDE | Power Difference Estimator |

# Chapter 1

# INTRODUCTION

## 1.1 OVERVIEW

Technology and system requirements in the telecommunications field are changing very fast. Over the prior years, considering that the transition from analog to digital communications, and from wired to wireless, different standards and solutions have now been adopted, developed, implemented and modified, often to cope with new and different business requirements. Today, more and more, telecommunication network operators struggle to offer new advanced services in a stylish and functional way [2].

Wireless communications [1]-[4] is just a rapidly growing bit of the communications manufacturing, with the potential to offer high-speed high-quality information exchange involving the portable devices located anywhere in the world. Potential applications enabled by this technology include multimedia Internet-enabled, Global System for Mobile (GSM), smart homes, automated highway systems, video teleconferencing and distance learning, and autonomous sensor networks, just to name a few. However, supporting these applications using wireless techniques creates a substantial technical challenge

The motion in space of an instant receiver operating in a multipath channel results in a communications link that experiences small-scale fading. The rapid fluctuations of the received power level because of small sub-wavelength changes in receiver position are referred to as small-scale fading [4]. Basically, mobile radio communication channels are time varying, multipath fading channels [3], [4]. In a radio communication system, there are numerous paths for a sign to feed from the transmitter to a receiver. Sometimes there is a direct path where in actuality the signal travels without being obstructed, which is called a Line Of Sight (LOS) path. Generally, aspects of the signal are refracted by different atmospheric layers or reflected by the bottom and objects involving the transmitter and the receiver such as for example vehicles, buildings, and hills, which is called Non Line Of Sight (NLOS) paths. These components travel in various paths of different length and combine at the receiver. Thus, signals on each path suffer different transmission delays and attenuation because of the finite propagation velocity. The combination of the signals at the receiver results in a destructive or constructive interference, depending on the relative delays involved. Actually, the surroundings changes as time passes leading to signal variation. This is called time variant environment. Also, the motion of the object influences signals. A short distance movement may cause an evident change in the propagation paths and vary the effectiveness of the received signals.

The popularity of OFDM is mainly due to the method of handling the multipath interference at the receiver. Multipath phenomenon generates two effects (a) Frequency selective fading and (b) Inter symbol interference (ISI).The "flatness" perceived with a narrowband channel overcomes the frequency selective fading. On the other hand, modulating symbols at a suprisingly low rate makes the symbols much more than channel impulse response and hence reduces the ISI. Utilization of suitable error correcting codes provides more robustness against frequency selective fading. The insertion of a supplementary guard interval between consecutive OFDM symbols can reduce the effects of ISI even more. The frequency spacing and time synchronization of the carriers is chosen in such a way that the carriers are orthogonal, meaning that they do not cause interference to each other.

OFDM is well suited for wideband, high data rate transmissions. The main advantage is that less equalization is required. A consequence of that is OFDM is not a very good solution for one to one communications with several users on shared channels, because of the problem of frequency allocation. However on super high frequency bands (SHF) and Extremely high frequency bands (EHF) where occupied bandwidth is not a great problem, OFDM may be a good solution for one to one communications. But, nowadays, OFDM is mainly used for one to many (broadcast) communications like radio or television broadcasting. That’s why we find OFDM on several new digital broadcasting systems such as DAB and DVB.

In spite of the success and effectiveness of the OFDM systems, it is suffering from two wellknown drawbacks: large Peak to Average Power Ratio (PAPR) and high sensitivity to Carrier Frequency Offset (CFO). The current presence of the CFO in the received carrier will lose orthogonality on the list of carriers and as the CFO causes a reduced total of desired signal amplitude in the output decision variable and introduces Inter Carrier Interference (ICI). After that it brings up an increase of Bit Error Rate (BER) [7]-[9]. The consequence caused by CFO for an OFDM/QAM system was analyzed in [8], and it absolutely was indicated that CFO should really be significantly less than 2% of the bandwidth of subchannel to guarantee the signal to interference ratio be higher than 30 dB.

## 1.2LITERATURE SURVEY

**P.H Moose [18] [1994]** Studied the effects of CFO on the performance of OFDM digital communications. He maintained SIR ratio of 20 dB or greater for the OFDM Carriers, the relative carrier frequency offset is limited to 4% or less of the inter carrier spacing.

**J.Vde Beek [19] [1997]** discussed a joint maximum likelihood (ML) symbol-time and carrier frequency offset estimator in OFDM systems. He also shows that the frequency estimator may be used in a tracking mode and the time estimator in an acquisition mode.

**W.L Chin [32] [2011]** Described a synchronization algorithm for determining the symboltiming offset and the carrier frequency offset (CFO) in OFDMsystems, based on the maximum-likelihood (ML) criterion.

**Yinsheng LIU [33] [2012]** Discussed a theoretical comparison based on Cramer Rao Bounds (CRB) for two kinds of CFO estimation methodsandshown that the performance of CFO estimation can be improved by exploiting the repetition property and the exact training symbol rather than exploiting the repetition property only.

**Morelli [34] [2013]** Proposed a joint ML estimator of CFO, noise power and SNR by exploiting the repetitive structure of a training preamble composed of several identical part.

## 1.2 MOTIVATION

The demand for future higher data rate communications always supplies the impetus for this research. It's obvious a parallel system is effective at carrying additional information when compared to a cascade system, simply because it uses a variety of frequency bands. However, the significant advantageous asset of OFDM is it is robust in frequency-selective channels, which result from either multipath fading or other communication interferences. The interference problem is severe specifically for some systems working in the Unlicensed National Information Infrastructure (UNII) operating frequency range, including the IEEE 802.11a standard. Under any of these conditions, the channel has non-uniform power gains, as well as nonlinear phases across frequencies.

A promising candidate (OFDM) that effectively mitigates performance degradations because of multipath and is effective at combating deep fades in area of the spectrum. The OFDM waveform may be easily modified to modify to the delay spread of the channel. OFDM are designed for large delay spreads easier to the independence of the carriers and the flexibility of varying the cyclic prefix length. OFDM allows efficient operation in both FDD and TDD mode as very short or no preambles are needed. Multiple access techniques which can be developed for the single carrier modulations (e.g. TDMA, FDMA) had made possible of sharing one communication medium by multiple amount of users. Multiple techniques schemes are used to allow many mobile users to share simultaneously a finite quantity of radio spectrum. The sharing is needed to achieve high capacity by simultaneously allocating the available bandwidth (or the available quantity of channels) to multiple users. For the quality communications, this must certanly be done without severe degradation in the performance of the system. FDMA, TDMA and CDMA are the well- known multiplexing techniques utilized in wireless communication systems.

While working together with the wireless systems using these techniques various problems encountered are (1) multi-path fading (2) time dispersion which lead to ISI (3) lower bit rate capacity (4) requirement of larger transmit power for high bit rate and (5) less spectral efficiency. Disadvantage of FDMA technique is its Bad Spectrum Usage. Disadvantages of TDMA technique is Multipath Delay spread problem. In an average terrestrial broadcasting, the transmitted signal arrives at the receiver using various paths of different lengths. Since multiple versions of the signal interfere with each other, it becomes difficult to extract the initial information. The utilization of orthogonal frequency division multiplexing (OFDM) technique provides better solution for the aforementioned mentioned problems. To be able to handle frequency selective fading, the transmitted OFDM signals are split into many sub channels so that those sub channels can be considered frequency flat approximately as the amount of the subchannel N is large enough. Hence the OFDM signals will suffer channel distortion less compared to conventional modulated signals.

Under OFDM modulation, the symbol duration becomes N times longer. For example, if the input data rate is 20Mbps, then, the symbol duration is 50 ns; however, in an OFDM system with 128 subcarriers, the symbol duration could become 6.4 microseconds. If these two kinds of symbols are modulated and transmitted by way of a channel with a particular rms value -say, rms = 60 ns, it's clear that the system with a longer symbol duration would perform better. In practice, the DVB-T standard suggests to make use of 2,048 subcarriers, or 8,192 subcarriers. In these cases, the symbol duration may be even increased by thousands of times.

## 1.3 STATEMENT OF PROBLEM

The estimation of the CFO is a classical problem, and it can be estimated via data added or nonblind algorithms, semi blind algorithms, and non-data added or blind algorithms. This thesis mainly concentrates on Maximum Likelihood (ML) CFO blind estimation algorithms based on data symbol repetition for OFDM systems. The scope of this work is as follows:

1. Investigation of-
   - ML CFO blind estimation algorithm based on cyclic prefix.
   - ML CFO blind estimation algorithm based on repetition of OFDM data symbols.
2. Propose a Modified. ML CFO blind estimation algorithm based on repetition of OFDM data symbols.
3. Estimation of CFO by using the various ML CFO blind algorithm.
4. Comparative study of various ML CFO blind algorithm in terms of mean square error (MSE).

### 1.4 THESIS ORGANIZATION

This thesis is organized into five chapters (including *Chapter 1*) as follows:

*Chapter* 1 provides a brief introduction about OFDM and CFO in OFDM system. This chapter also includes the reason (under Motivation) that leads to this report followed by its contribution in wireless communication.

*Chapter* 2 presents basics of OFDM including transmitter and receiver. It also includes advantages, disadvantages and applications of OFDM in today's wireless communication.

*Chapter* 3 gives the brief overview of ICI and its mathematical modeling. This chapter also presents various Blind CFO estimation techniques based on symbol repetition and based on cyclic prefix.

*Chapter 4* presents a Modified Blind ML CFO estimation technique based on data symbol repetition and also gives simulation results along with comparisons.

*Chapter 5* concludes the thesis.

# Chapter 2

# Orthogonal Frequency Division Multiplexing

## 2.1OVERVIEW OF OFDM SYSTEM

As wireless communication evolves towards broadband systems to support high data rate applications, we need a technology that may efficiently handle frequency selective fading. The Orthogonal Frequency Division Multiplexing (OFDM) system is widely found in this context. The main element notion of OFDM would be to divide the whole transmission band into several parallel subchannels (also called subcarriers) so that each subchannel is just a flat fading channel [11]-[13]. In cases like this, channel equalization can be performed in most subchannels in parallel using simple one-tap equalizers, which may have really small computational complexity.

A block diagram of an OFDM system is depicted in *fig.2.1.1*.Here, for simplicity and clearness of illustration, channel coding block is left out. The incoming digital data are first passed to a serial to parallel converter (S/P) and changed into blocks of $N$ data symbols. Each block is called a frequency-domain OFDM symbol and $N$ is the number of sub channels.

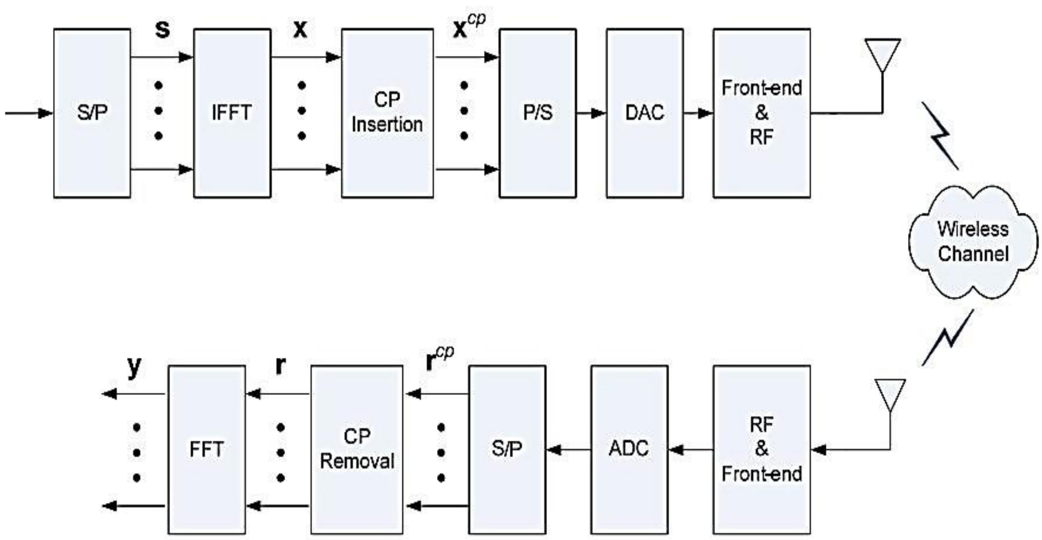


*Figure 2. 1.1 Block diagram of an OFDM system* [2]

Let us use $s = [s_0, s_1, \dots \dots .. s_{N-1}]^T$, where subscript $T$ denotes vector transpose, to denote one frequency domain OFDM symbol. The modulation in OFDM is performed using the Inverse Discrete Fourier Transform (IDFT) as follows.

$$\mathbf{x} = \mathbf{Ws}, \tag{2.1.1}$$

Where $\mathbf{W}$ denotes the $N \times N$ IDFT matrix, with the $(m,n)^{\text{th}}$ element given by

$$W_{m,n} = \frac{1}{\sqrt{N}} exp(j2\pi)\frac{mn}{N}$$

In practice, the IDFT is normally performed using a more computationally efficient method, the Inverse Fast Fourier Transform (IFFT). After modulation, the last $Ng$ samples of $\mathbf{x}$ are appended in front of$\mathbf{x}$, such that

$\mathbf{x}^{\text{cp}} = [x_{N-N_g}, x_{N-N_g+1}, \dots \dots \dots .. x_{N-1}, x_0,\ x_1, \dots \dots \dots x_{N-1,}]^{\boldsymbol{T}}$ is cyclic. These $N_g$ samples are Called cyclic prefix (CP) and $\mathbf{x}^{\text{cp}}$is called time domain OFDM symbol. The process of CP insertion can be written in an equivalent matrix form as $\mathbf{x}^{\text{cp}} = \mathbf{A}^{\text{cp}}\mathbf{x}$, where $\mathbf{A}^{\mathbf{cp}} = [\mathbf{I}_{\text{N}}(N - N_g : N - 1, :); \mathbf{I}_{\text{N}}]$.Here, $\mathbf{I}_{\text{N}}$denotes the identity matrix of size $N \times N$. After CP insertion, the Time-domain OFDM symbol $\mathbf{x}^{\text{cp}}$ is passed to a parallel to serial converter (P/S). The output is converted to an analog signal using a digital to analog converter (DAC), modulated and amplified through the front-end and radio frequency (RF) block and transmitted via the antenna to the wireless channel.

At the receiver, the received RF signal at the receive antenna is first demodulated through the receiver RF and front-end block. The resulting analog signal is then converted to digital form using the analog to digital converter (ADC) and then the serial digital signal is converted to time domain symbols $\mathbf{r}^{\text{cp}}$ of the size $N+N_g$through the serial to parallel converter (S/P). Considering the transmission of only current OFDM symbol$\mathbf{x}^{\text{cp}}$, the $k^{th}$ sample of $\mathbf{r}^{\text{cp}}$ can be written as

$$r_k^{cp} = \sum_{i=0}^{L-1} h_{k-i}\, x_i^{cp} + n_k, \tag{2.1.2}$$

Where $h_k$ is the $k^{th}$ tap of the impulse response of the multipath channel $\mathbf{h} = [h_0, h_1, \dots \dots \dots h_{L-1}]^T$, $x_i^{cp}$ is the $i^{\text{th}}$ element of $\mathbf{x}^{\text{cp}}$ and $n_k$is the AWGN.

Here $L$ is used to denote the maximum length of channel impulse response. To make sure that there is no ISI the length of cyclic prefix should satisfy $N_g \geq L$. Using matrix notation the received signal in *eq.2.1.2* can be written equivalently as

$$\mathbf{r}^{cp} = \mathbf{H}_t \mathbf{x}^{cp} + \mathbf{n}, \qquad (2.1.3)$$

Where $\mathbf{H}_t$is a $(N + N_g) \times (N + N_g)$ lower triangular Toeplitz matrix with the first column given

by$[h_0, h_1, \dots \dots \dots h_L, 0 \dots \dots ., 0]^T$ as shown below

$$\mathbf{H}_t = \begin{bmatrix} h_0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ h_1 & h_0 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ h_{L-1} & h_{L-2} & \cdots & h_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & h_0 & 0 \\ 0 & 0 & \cdots & 0 & \cdots & h_1 & h_0 \end{bmatrix}.$$

At the receiver, the first $N_g$ samples of $\mathbf{r}^{cp}$ due to the cyclic prefix are removed, which is indicated by the CP removal block in *fig.2.1.1*. Again this can be written in matric form as $\mathbf{r} = \mathbf{D}^{cp}\mathbf{r}^{cp}$, where $\mathbf{D}^{cp} = [\mathbf{0}_{N \times N_g}, \mathbf{I}_g]$ with $\mathbf{0}_{N \times N_g}$denotes a matrix of size $(N + N_g)$ whose elements are all zero. Hence, we have the received time domain signal after CP removal given by

$$\mathbf{r} = \mathbf{D}^{cp}\mathbf{H}_t\mathbf{A}^{cp}\mathbf{Ws} + \mathbf{n} = \mathbf{H}_c\mathbf{Ws} + \mathbf{n}, \qquad (2.1.4)$$

Where $\mathbf{H}_c = \mathbf{D}^{cp}\mathbf{H}_t\mathbf{A}^{cp}$**.** Notice that the effects of CP insertion, channel convolution and CP removal are combined into a single matrix$\mathbf{H}_c$. It can be easily shown that $\mathbf{H}_c$ is a $N \times N$ circulant matrix given by

$$\mathbf{H}_c = \begin{bmatrix} h_0 & 0 & \cdots & 0 & \cdots & h_2 & h_1 \\ h_1 & h_0 & \cdots & 0 & \cdots & h_3 & h_2 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ h_{L-1} & h_{L-2} & \cdots & h_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & h_0 & 0 \\ 0 & 0 & \cdots & 0 & \cdots & h_1 & h_0 \end{bmatrix}.$$

Next the time-domain signal is transformed to the frequency-domain using an $N$-point FFT. The frequency domain received signal can be written as

$$\boldsymbol{y} = \mathbf{W}^H\mathbf{r} = \mathbf{W}^H\mathbf{H}_c\mathbf{W}\mathbf{s} + \mathbf{W}^H\mathbf{n},$$

Where $\mathbf{W}^H$ is an $N \times N$ DFT matrix and superscript $^H$ denotes matrix Hermitian. Since $\mathbf{H}_c$ is a circulant matrix, it can be diagonalized by the IDFT matrix as follows

$$\mathbf{H}_c = \mathbf{W}\mathbf{H}\mathbf{W}^H,$$

Where $\mathbf{H}$ is diagonal matrix given by $\mathbf{H} = \mathrm{diag}(\mathbf{W}^H\mathbf{h}_c)$ and $\mathbf{h}_c$ is the first column of $\mathbf{H}_c$. In other words, the diagonal elements of $\mathbf{H}$ are the DFT of the channel impulse response $\mathbf{h}$ and can be interpreted as the channel frequency responses on $N$sub channels. Using this property, the frequency-domain received signal can be written as

$$\begin{aligned} \boldsymbol{y} &= \mathbf{W}^H\mathbf{H}_c\mathbf{W}\mathbf{s} + \mathbf{W}^H\mathbf{n} = \mathbf{W}^H(\mathbf{W}\mathbf{H}\mathbf{W}^H)\mathbf{W}\mathbf{s} + \mathbf{W}^H\mathbf{n} \\ &= \mathbf{H}\mathbf{s} + \mathbf{n}', \end{aligned} \tag{2.1.5}$$

Where $\mathbf{n}'$ is the frequency-domain noise term, which is also Gaussian distributed with zero mean and has the same variance as $\mathbf{n}$. Because $\mathbf{H}$ is a diagonal matrix, so different subcarriers are perfectly decoupled after the FFT operation and the frequency selective fading channel can be equalized using a simple one-tap equalizer on each subcarrier individually.

By means of illustration, the amplitude spectra of subcarriers 6 to 10 for an OFDM system with $N = 16$ are sketched in *fig.2.1.2* [11]. It may be seen that the spectra of different subcarriers are overlapping. At the center of each subcarrier, the signals from another subcarriers are 0. Which means that in OFDM systems, different subcarriers are orthogonal at the center of each subcarrier, although their spectra are overlapping?

From above, it can be seen that in OFDM systems, the frequency selective fading channel is divided in to a number of flat fading sub channels. As a result, complicated time-domain equalization of the frequency selective fading channel can be performed equivalently in the frequency-domain utilizing a simple one-tap equalizer on each sub channel. Hence, OFDM supplies a better method to take care of frequency selective fading compared to single carrier systems as time passes domain equalizer.

By combining OFDM with error control coding, the coded OFDM system can also be better made to narrow band interferences [11]. This is because narrow band interferences only affects a small number of subcarriers and causes detection errors on these subcarriers. These detection errors can usually be corrected by error control coding. Due to these advantages, OFDM has been adopted in lots of modern wireless communication standards such as IEEE 802.11a/g WLAN, IEEE 802.16e Broadband Wireless Access (also known as WiMAX), Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB).

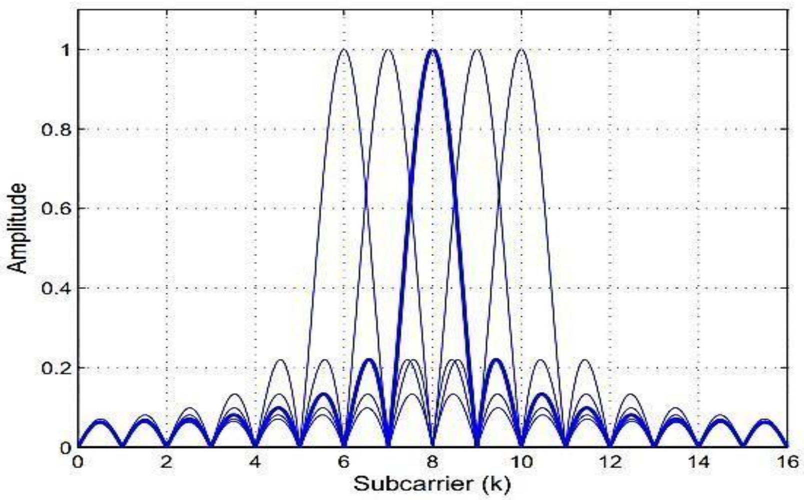


*Figure 2.1.2Amplitude spectra of subcarriers 6 to 10 for an OFDM system with 16* subcarriers.[11]

However, OFDM also has some disadvantages. Firstly, because the modulation is performed using IDFT, the peak to average power ratio (PAPR) of time domain OFDM signals is higher compared to single carrier systems. This puts high requirements on the dynamic range of the RF amplifiers and introduces extra clipping noise in the system [12], [13]. Another disadvantage of the OFDM system is that it is more sensitive to frequency synchronization errors compared to single carrier systems.

## 2.2EFFECTS OF FREQUENCY SYNCHRONIZATION ERRORS IN OFDM SYSTEM

In the earlier section, an breakdown of OFDM systems is presented. Features of OFDM were highlighted and plus it is found that sensitivity to frequency synchronization errors in the shape of carrier frequency offset (CFO), is really a key disadvantage of OFDM systems. In this section, a more descriptive study on the results of CFO on the performance of OFDM systems is presented. While the name suggests, CFO is definitely an offset between the carrier frequency of the transmitted signal and the carrier frequency used at the receiver for demodulation. In wireless communications, CFO comes mainly from two sources:

- The mismatch between oscillating frequencies of the transmitter and the receiver local oscillators (LO).
- The Doppler effect of the channel due to relative movement between the transmitter and the receiver.

At the receiver, the effect of CFO is mitigated through frequency synchronization. *Fig.2.2.1* shows an OFDM receiver with frequency synchronization implemented in both the analog and the digital domains. The received signal from the receive antenna is first passed to the receiver front-end. Here, to make sure that the local oscillator at the receiver front-end is operating with sufficient accuracy, its reference frequency is continuously adjusted by the analog coarse frequency synchronization unit [13], which is made

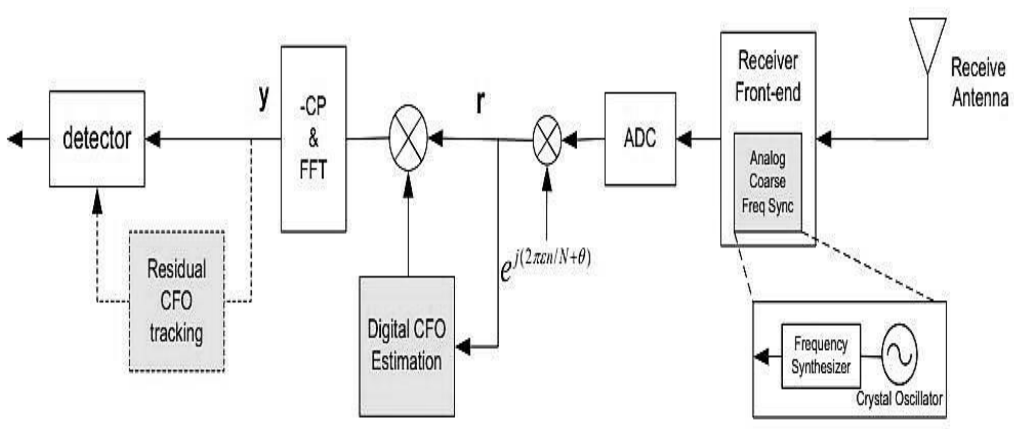


*Figure 2.2.1An OFDM receiver with frequency synchronization.*[13]

up of crystal oscillator and a volume synthesizer.

The specifications for the worst-case frequency errors for both transmitter and receiver LOs (crystal oscillator and frequency synthesizer) are 20 ppm (parts per million). This leads to a worst-case CFO of 96 kHz (40 ppm) for center frequency of 2.4 GHz after analog coarse frequency synchronization. For WLAN applications, the maximum duration of a data packet is in the order of *ms* and the variation of the LO output frequency within this short time duration is negligible. Therefore, the digital domain CFO after analog frequency synchronization can be considered a constant value and estimated once per data packet. After the analog to digital converter, we denote the digital domain CFO normalized with respect to the subcarrier spacing of the OFDM system as $\varepsilon$. This CFO introduces a time dependent phase rotation $e^{j(2\pi\varepsilon n/N)}$ to the received digital time-domain signal, where $n$ is the time index, and $N$ is the number of subcarriers. Together with a constant phase offset $\theta$ due to the channel and the analog processing, this introduces a phase rotation of $e^{j(\frac{2\pi\varepsilon n}{N}+\theta)}$as shown in *fig.2.2.1*. In this way, we can write the received time-domain signal in the $m^{\text{th}}$ OFDM symbol interval in the following form [16]

$$\mathbf{r}^m = \mathbf{E}\mathbf{W}\mathbf{H}^m\mathbf{s}^m e^{j(2\pi\varepsilon(m-1)\left(1+\frac{N_g}{N}\right)+\theta)} + \mathbf{n}^m. \tag{2.2.1}$$

The CFO matrix $\mathbf{E} = \mathrm{diag}(1, e^{\frac{j2\pi\varepsilon}{N}}, \ldots\ldots e^{j2(N-1)\varepsilon/N})$ is a diagonal matrix containing the CFO value $\varepsilon$, and $N$ is the number of subcarriers. Matrix $\mathbf{W}$is the$N \times N$ IDFT matrix, $\mathbf{H}^m$is a channel frequency response for different subcarriers, $\mathbf{s}^m$is the transmitted signal for the $m^{th}$OFDM symbol and $\mathbf{n}^m$ is the AWGN noise vector. Here we split the phase rotation caused by the CFO into the CFO matrix $\mathbf{E}$ and a phase offset $e^{j(2\pi\varepsilon(m-1)\left(1+\frac{N_g}{N}\right))}$for OFDM symbol $m$.

Notice from *eq.2.2.1* that the effects of the CFO $\varepsilon$ and the constant phase offset $\theta$ are represented in the following three terms:

1. A constant phase offset$e^{j\theta}$.
2. A CFO matrix$\mathbf{E}$.
3. A CFO and OFDM symbol index ($m$) dependent phase offset

$$e^{j(2\pi\varepsilon(m-1)\left(1+\frac{N_g}{N}\right))}$$

The constant phase offset $e^{j\theta}$ is a common scalar multiplied with all the received signals. This gives the same phase offset of $e^{j\theta}$ on all the frequency domain received signals. In this way, it can be considered as part of the frequency domain channel and can be estimated together with the frequency domain channel and compensated using one-tap equalizers. However, the CFO ($\varepsilon$) needs to be estimated and compensated in enough time domain. This is because; CFO introduces inter-carrier interference (ICI) in the frequency-domain received signals. For IEEE 802.11g systems, the worst-case digital domain CFO of 96 kHz corresponds $\varepsilon$o = 0.31. The ability of ICI for this reason CFO is significantly larger than that of the AWGN noise. This makes CFO estimation in the frequency domain much more difficult compared to that particular in enough time domain since the signal to interference ratio in the frequency domain is really low because of the large ICI. In *fig.2.2.1*, the time-domain CFO estimation is performed in the digital CFO estimation block. The aftereffect of the CFO is compensated from the received signal utilizing the estimate. The compensated signal is passed through the – CP (CP removal) & FFT block and is transformed to the frequency domain. The frequency-domain signal is then passed to the detector.

As shown in *fig.2.1.2*, in OFDM systems, orthogonality between different subcarriers is maintained only if sampling occurs at the correct frequency, i.e. in the biggest market of each sub channel. *Fig.2.2.2* illustrates what goes on if you find an optimistic CFO. Firstly, the amplitude of the required signal is attenuated. Secondly, the orthogonality between different subcarriers is destroyed and on the required subcarrier, there exists non-zero ICI from all of those other subcarriers.

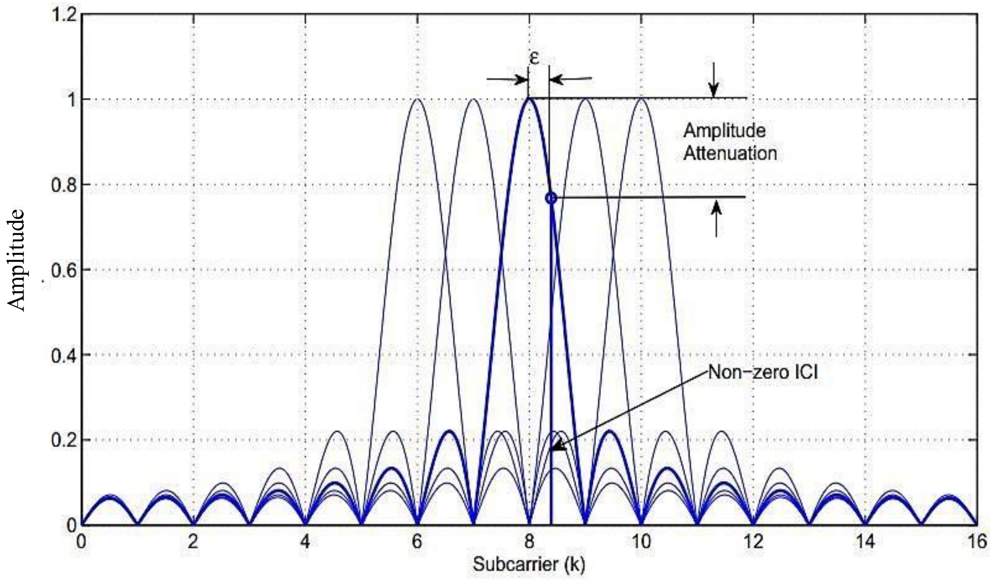


*Figure 2.2.2* Effects of CFO in OFDM [11]

From *eq.2.2.1*, we can rewrite each element of $\mathbf{r}$ in summation form as

$$r_k = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} H_l s_l \exp\left(j2\pi \frac{(l+\varepsilon)k}{N}\right) + n_k, \qquad (2.2.2)$$

Where $H_l$ and $s_l$ are the channel response and transmitted signal on the $l^{th}$ subcarrier respectively.

Here we omit the constant phase offset $e^{j\theta}$ because it can be considered as part of the channel response. Moreover, as the length of the CP is larger than the length of the channel impulse response, there is no ISI between different OFDM symbols. Hence, the OFDM symbol index *m* in *eq.2.2.2* is not important for the analysis and is also dropped. Taking the FFT of the received signal $\mathbf{r}$, we get the received signal on the $l^{th}$ subcarrier as

$$
\begin{aligned}
y_l &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r_k \exp\left(-\frac{j2\pi kl}{N}\right) \\
&= \frac{1}{N} \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} H_i s_i \exp\left(-\frac{j2\pi k}{N}(l-i-\varepsilon)\right) + n_l' \\
&= \sum_{i=0}^{N-1} H_i s_i \exp\left[j\pi(i-l+\varepsilon)\left(1-\frac{1}{N}\right)\right] \frac{\sin\left(\pi(i-l+\varepsilon)\right))}{N\sin\left(\frac{\pi(i-l+\varepsilon)}{N}\right)} + n_l' \\
&= \left\{\frac{\sin(\pi\varepsilon)}{N\sin\left(\frac{\pi\varepsilon}{N}\right)} \exp(j\pi\varepsilon(1-1/N))\right\} H_l s_l + I_l + n_l',
\end{aligned}
$$

(2.2.3)

where $I_l$ is the inter-carrier interference from all the other subcarriers on sub-carrier $l$ given by

$$
I_l = \sum_{k=0,k\neq l}^{N-1} H_k s_k \exp\left[j\pi(k-l+\varepsilon)\left(1-\frac{1}{N}\right)\right] \frac{\sin\left(\pi(k-l+\varepsilon)\right)}{N\sin\left(\frac{\pi(k-l+\varepsilon)}{N}\right)},
$$

(2.2.4)

and $n_l'$ is the AWGN noise in the frequency domain with variance $\sigma_n^2$. *Eq.2.2.3* gives the mathematical description of the two detrimental effects of the CFO in OFDM systems. Firstly the amplitude of the desired signal is attenuated to $\frac{sin\pi\varepsilon}{Nsin(\pi\varepsilon/N)} < 1$. Secondly, besides AWGN noise $n_l'$, there is an additional ICI term $I_l$. In this case, the signal to interference and noise ratio (SINR) of the received signal on subcarrier $l$ is given by

$$
SINR_l = \frac{\mathrm{E}(|\mathrm{H}_l|^2)\mathrm{E}(|s_l|^2)sin^2(\pi\varepsilon)/N^2 sin^2\left(\frac{\pi\varepsilon}{N}\right)}{\mathrm{E}(|I_l|^2) + \sigma_n^2}. \quad (2.2.5)
$$

As the ICI power is independent of the AWGN noise power, the ICI causes more degradation in high SNR cases compared to low SNR cases. Therefore, to guarantee good performance of OFDM systems, CFO must be accurately estimated and compensated.

### 2.3 OFDM MERITS, DEMERITS & APPLICATIONS

The OFDM is a promising transmission scheme, which has been considered extensively, as it has the following key advantages [5]-[13]:

- OFDM makes efficient use of the spectrum.

- OFDM becomes more resistant to frequency selective fading than single carrier systems by converting the frequency selective fading channel into narrowband flat fading sub channels.
- OFDM eliminates Inter Symbol Interference (ISI) and Inter Frame Interference (IFI) through use of a Cyclic Prefix (CP).
- OFDM recovers the symbols lost due to the frequency selectivity of the channel by using adequate channel coding and interleaving.
- OFDM makes channel equalization simpler than single carrier systems by using adaptive equalization techniques.
- OFDM seems to be less sensitive to sample timing offsets in comparison with single carrier systems.
- OFDM provides good protection against co-channel interference and impulsive parasitic noise.
- OFDM makes it possible to use Maximum Likelihood (ML) decoding with reasonable complexity. OFDM is computationally efficient with FFT techniques.

The several advantages of the OFDM systems could only appear if the main three drawbacks were treated carefully. OFDM has the following negative aspect:

- OFDM signal has a noise like amplitude with a very large dynamic range; therefore, it requires RF power amplifiers with a high peak to average power ratio, which may require a large amplifier power back off and a large number of bits in the Analog to Digital (A/D) and Digital to Analog (D/A) designs.
- OFDM is very sensitive to Carrier Frequency Offset (CFO) caused by Doppler Effect. Hence, CFO should be estimated and cancelled completely.
- OFDM receiver suffers from the difficulty to make a decision about the starting time of the FFT symbol.

We have seen that OFDM is digital transmission technique well suited for wideband, high data rate transmissions. The main advantage is that less equalization is required. A consequence of that is OFDM is not a very good solution for one to one communications with several users on shared channels, because of the problem of frequency allocation. However on super high frequency bands (SHF) and Extremely high frequency bands (EHF) where occupied bandwidth is not a great problem, OFDM may be a good solution for one to one communications. But, nowadays, OFDM is mainly used for one to many (broadcast) communications like radio or television broadcasting. That's why we find OFDM on several new digital broadcasting systems such as DAB and DVB.

*Digital Audio Broadcasting (DAB):*

Digital Audio Broadcasting (DAB) is an international, standardized digital broadcasting system developed by the European EUREKA-147 Project. The system should completely replace, in the future, the well-known analog FM (Frequency Modulation) radio system on the 88-108 MHz frequency band. The DAB system is digital and provides CD-like audio quality. DAB is much more robust to interferences and is well suited for mobile reception like in a car. New possibilities are available on receivers like for example multimedia features (image and texts). The transmission scheme used for DAB is OFDM modulation. DAB is being deployed around Europe and some other countries now a day, receivers are still expensive like CD players in the beginning of eighties but we might expect to see a wide use of it in the next 5 years.

***ADSL:***

Asymmetric Digital Subscriber Line (ADSL) is a technique to transmit high data rates (up to 6 Mb/s downlink, 640kb/s uplink) on Subscriber Lines (telephone lines). Such lines consist of twisted copper wires. The idea is to use the full capacity of the line instead of using only 4 kHz needed to transmit voice. Occupied bandwidth goes to 1.1 MHz .The main problem is that the characteristics of the line change among users. They change with distance, presence of bridged taps in the line, neighborhood of other lines. The results are reflections at certain frequencies which cause attenuation, velocity dependent of the frequency which causes ISI. The situation is very similar to wireless channels.

There are 2 possible modulation schemes usable for ADSL: CAP (Carrier less amplitude phase) that is similar to QAM and Discrete Multi-tone (DMT) that is another appellation for OFDM. Nowadays, it seems that DMT is the retained candidate for ADSL. The downlink consists of 222 tones (carriers) and uplink is spitted in 24 tones. 2 to 15 bits are coded by tone. The transmission rate is optimized with respect to line conditions. If transmission on one of the tone is disrupted because of strong reflections and interferences at the frequency band, transmission is suspended on that tone by modem.

# Chapter 3

# ANALYSIS OF INTER-CARRIER INTERFERENCE

## 3.1INTRODUCTION

OFDM is an excellent technique to manage impairments of wireless communication channels such as for instance multipath propagation. Hence, OFDM is an operating candidate for future 4G wireless communications techniques [1]-[4]. On one other hand, one of numerous major drawbacks of the OFDM communication system could be the drift in reference carrier. The offset within received carrier will miss orthogonality on the list of carriers. Hence, the CFO causes a diminished number of desired signal amplitude in the output decision variable and introduces ICI. Then it raises a rise of BER. The result attributable to CFO for OFDM system was analyzed in [7]-[9]. In [7] BER upper bound of OFDM system is analyzed without ICI self-cancellation and BER of OFDM system is analyzed using self-cancellation, but this technique is less accurate. In [9], it's indicated that CFO must certanly be less than 2% of the bandwidth of the sub channel to guarantee the signal to interference ratio to be greater than 30 db. A critically sampled OFDM/OQAM system could be not robust to CFO [9], even when optimal pulses are employed as shaping filters. Thus, carrier frequency offset greatly degrades system performance. Therefore, practical OFDM systems need the CFO to be compensated with sufficient accuracy, and it has generated plenty of literature on CFO estimation algorithms.

Most of the prevailing CFO estimators for OFDM derive from periodically transmitted pilot symbols. Yet, the pilot symbols transmission loses an important bandwidth, especially in the case of continuous transmissions. Therefore, pilot-based schemes are mainly fitted to packet oriented applications.

Semi blind approaches proposed in the literature will be the first faltering step to boost the bandwidth efficiency [20]. Those usually depend on various assumptions such as having an individual pilot symbol, two identical consecutive OFDM data blocks, or some specific structure within the OFDM symbol.

Recently, blind, or non-data aided have received extensive attention, whilst the bandwidth will undoubtedly be kept without a doubt data. Among different classes of blind methods, subspace based methods [18]-[20] could be the famous category which were lately demonstrated to be comparable to ML estimator [19]. Those methods is determined by the reduced rank signal model induced by either some unmodulated carriers or virtual carriers (VC) at the edges of OFDM block, which aim at minimizing the interference caused to adjacent OFDM systems. While OFDM systems are suited by design to multipath transmission, many existing CFO estimators deal only with frequency flat channels. Extension of ML solutions to multipath Rayleigh fading channels may be within [21]. Now, non-circularity introduced by real-valued modulations was exploited in [22]. In [23] a blind CFO estimation algorithm has been derived by exploiting the conjugate second-order cyclostationarity of the received OFDM signal in the case of noncircular transmission. In [24] this approach, designed for standard OFDM systems, has been extended and analyzed in the context of OFDM/OQAM transmissions. On the other hand, the derived estimator assures adequate performance only every time a large amount of OFDM symbols are considered. In [25] a blind joint CFO and symbol timing estimator on the foundation of the unconjugated cyclostationarity property of the OFDM/OQAM signal has been derived. Constant modulus (CM) constellations allow highly accurate CFO estimation [26].

In the blind CFO Estimator the used sub channels will be totally used to transmit real data and the CP will not be extended by any extra guard intervals. The blind estimators are thought as a bandwidth efficient ones. The blind estimators of the CFO in the OFDM system may be built basically on the foundation of the structure of the OFDM frame or its components: Blind CFO estimators on the foundation of the used carriers, VC based blind CFO estimators, and the CP based blind CFO estimators. In these subsections different blind estimators centered on used carries are introduced.

## 3.2 ICI MECHANISM OF STANDARD OFDM SYSTEMS

The main disadvantage of OFDM is its susceptibility to small differences in frequency at the transmitter and receiver, normally referred to as frequency offset. This frequency offset may be caused by Doppler shift due to relative motion involving the transmitter and receiver, or by differences involving the frequencies of the neighborhood oscillators at the transmitter and receiver.

In this thesis, the frequency offset is modeled as a multiplicative factor introduced in the channel, as shown in *fig.3.2.1*.

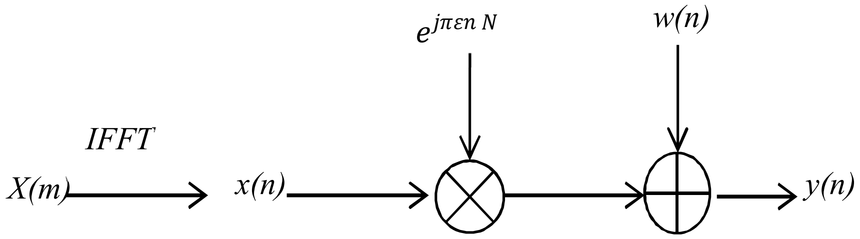


*Figure 3.2.1 Frequency Offset Model.*

The received signal is given by,

$$y(n) = x(n)\exp\left(\frac{j2\pi n\varepsilon}{N}\right) + w(n) \qquad (3.2.1)$$

Where $\varepsilon$ is the normalized frequency offset, and is given by $\Delta f N T_s$, $\Delta f$ is the frequency difference between the transmitted and received carrier frequencies and $T_s$ is the subcarrier symbol period. *w(n)* is the AWGN introduced in the channel.

The effect of this frequency offset on the received symbol stream can be understood by considering *Y(k)* the received symbol on the $k^{th}$ sub-carrier.

$$Y(k) = X(k)S(0) + \sum_{l=0, l\neq k}^{N-1} X(l)S(l-k) + n_k \qquad (3.2.2)$$

*k = 0,1,2 ……. N-1*

Where *N* is the total number of subcarriers, *X(k)* is the transmitted symbol(M-ary Phase Shift Keying) for $k^{th}$ subcarrier, $n_k$ is FFT of *w(n)* and *S(l-k)* are the complex coefficients for ICI components in the received signal. The ICI components are the interfering signals transmitted on sub-carriers other than $k^{th}$ subcarrier. The complex coefficients are given by

$$S(l-k) = \frac{\sin(\pi(l+\varepsilon-k))}{N\sin(\pi(l+\varepsilon-k)/N)}\exp(j\pi\left(1-\frac{1}{N}\right)(l+\varepsilon-k) \qquad (3.2.3)$$

The carrier-to-interference ratio (CIR) is the ratio of the signal capacity to the energy in the interference components. It indicates the potency of signal quality. CIR has been derived from eq.3.2.2 and is given below. The derivation assumes that the standard transmitted data has zero mean and the symbols transmitted on different subcarriers are statistically independent. In deriving the theoretical CIR expression, the additive noise is omitted. The desired received signal power on the $k^{th}$ sub carrier can be represented as

$$E[|C(k)|^2] = E[X(k)S(0)^2] \tag{3.2.4}$$

The ICI power can be represented as

$$E[|I(k)|^2] = E\left[\left|\sum_{l=0,l\neq k}^{N-1} X(l)\, S(l-k)\right|^2\right] \tag{3.2.5}$$

CIR is given as

$$CIR = \frac{S(k)^2}{\sum_{l=0,l\neq k}^{N-1} S(l-k)^2} = \frac{|S(0)^2|}{\sum_{l=1}^{N-1} S(l)^2} \tag{3.2.6}$$

*Factors Inducing ICI:*

ICI is distinctive from the co-channel interference in MIMO systems. The co-channel interference is a result of reused channels in other cells [3], while ICI results from an added sub-channels in the exact same data block of the exact same user. Although only 1 user is in communication, ICI might occur, yet the co-channel interference will not happen. There are two factors that cause the ICI, namely frequency offset and time variation. As discussed in [14], some types of time variations of channels could be modeled as a white Gaussian random noise when N is big enough, while other time variations could be modeled as frequency offsets, such as for instance as an example Doppler shift. Only frequency-offset is discussed in this chapter. ICI problem would become more complicated once the multipath fading is present.

*Doppler Effect:*

The relative motion between receiver and transmitter, or mobile medium one of them, would result in the Doppler Effect, a volume shift in narrow-band communications. For instance, the Doppler Effect would influence the caliber of a cell phone conversation in a moving car.

Generally speaking, the Doppler frequency shift may be formulated in a function of the relative velocity, the angle between the velocity direction and the communication link, and the carrier frequency, as shown in fig.3.2.2

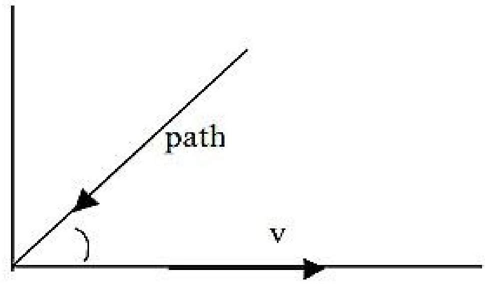


*Figure 3.2. 2 Doppler Effect* [4]

The value of Doppler shift could be given as

$$f_d = \frac{v}{\lambda} 2\pi \cos(\theta), \tag{3.2.7}$$

Where $\theta$ is the angle between the velocity and the communication link, which is the generally modeled as a uniform distribution between 0 and $2\pi$, $v$ is the receiver velocity, and $\lambda$ is the carrier wavelength.

Let us assume that electromagnetic wave velocity is C, the wavelength of carrier can be written as

$$\lambda = \frac{C}{f_c}, \tag{3.2.8}$$

Where $f_c$ is the carrier frequency,

Three kinds of Doppler Effect models are generally discussed in existing literatures: the classical model, the uniform model, and the two-ray model. The classical model can also be referred as Jake model, which was proposed in 1968 [15]. In this model, the transmitter was assumed to be fixed with vertically polarized antenna. There is no Light of Sight (NLOS) path, and all path gains were susceptible to identical statistics. It have been proved in 1972 [4] that the spectrum of this type of Doppler shift could be provided with as:

$$P_j(f) = \frac{k}{\pi f_m \sqrt{1 - \left(\frac{f - f_c}{f_m}\right)^2}}, \tag{3.2.9}$$

Where $k$ was fixed constant to given channel and antenna, $f_m$was the maximum Doppler shift as in *eq.3.2.7* and $f_c$ was defined in *eq.3.2.8.*

The uniform model is much simpler. Both velocity and angle are supposed to be uniformly distributed [16]. In this case, the power spectrum could be written as:

$$P_u(f) = \begin{cases} \frac{1}{2f_d}, & |f_d| \leq f_d \\ 0, & otherwise \end{cases} \tag{3.2.10}$$

The two-ray model assumed that there were only two paths between the transmitter and receiver. Accordingly, the resulting power spectrum is given as [16]

$$P_T(f) = \frac{1}{2}[\delta(f - f_d) + \delta(f + f_d)] \tag{3.2.11}$$

## 3.3BLIND CFO ESTIMATION USING REPETITION OF OFDM SYMBOLS

An OFDM transmission symbol is given by the $N$ point complex modulation sequence

$$x_n = \left(\frac{1}{N}\right) \sum_{k=-K}^{K} X_k \, e^{\frac{2\pi jnk}{N}}; \qquad n = 0,1,2, \dots\dots N-1; \qquad N \geq 2K+1 \tag{3.3.1}$$

It consists of *2K+1* complex sinusoids which have been modulated with *2K+1* complex modulation values $\{X_k\}$. We note that the individual sinusoids are orthogonal on the symbol interval, that is

$$\sum_{n=0}^{N-1} x_{nk}\, x_{nl}^{\,*} = \left(\frac{1}{N}\right) |X_k|^2 \delta_{kl} \qquad (3.3.2)$$

Where

$$x_{nk} = \left(\frac{1}{N}\right) X_k e^{\frac{2\pi jnk}{N}}.$$

We also note that the *N* point DFT of *eq.3.3.1* is the *N* point sequence Of modulation values.

$$DFT\{x_n\} = \left\{\sum_{n=0}^{N-1} x_n\, e^{-\frac{2\pi jnk}{N}}\right\}$$

$$= \{X_0, X_1, \ldots \ldots., 0,0, \ldots \ldots 0, X_{-K}, \ldots \ldots ..X_{-2}, X_{-1}\} \qquad (3.3.3)$$

*Eq.3.3.1* is the IDFT of *eq.3.3.3* and defines a practical modulation carrier synthesis technique for generating OFDM with perfect orthogonality.

After passing through a bandpass channel, the complex envelope of the received sequence can be expressed as

$$y_n = \left(\frac{1}{N}\right)\left[\sum_{k=-K}^{K} X_k H_k\, e^{\frac{2\pi jn(k+\varepsilon)}{N}}\right] + w_n;$$

$$n = 0,1,2,\ldots\ldots\ldots..,N-1 \qquad (3.3.4)$$

Where $H_k$ is the transfer function of the channel at the frequency of the $k^{th}$ subcarrier, $\varepsilon$ is the relative frequency offset of the channel (the ratio of the actual frequency offset to the intercarrier spacing), and $w_n$ is the complex envelope of the AWGN. Let the actual symbol transmitted be the $N+N_g$ point sequence

$$\left\{x_{N-N_g}, \ldots \ldots \ldots, x_{N-2}, x_{N-1}, x_0, x_1, \ldots \ldots \ldots x_{N-1}\right\} \qquad (3.3.5)$$

With $N_g$ greater than or equal to the time spread of the channel. The $N_g$ point precursor signal allows the received state by n = 0 (we assume synchronization at this stage of the receiver) leading to a received sequence as given by *eq.3.3.4*. It is assumed that the impulse response of the channel does not change (much) during the symbol plus guard interval (this corresponds to “slow-fading” in a radio frequency channel).

The insertion of guard intervals renders the received carriers orthogonal on the N point symbol interval. However, the demodulation process, which is implemented with a DFT (the DFT is equivalent to matched filter reception in the absence of frequency offset) is affected by frequency offset. That is,

$$Y_k = \sum_{n=0}^{N-1} y_n e^{-2\pi jkn/N}, \tag{3.3.6}$$

The $k^{th}$ element of the DFT sequence consists of three components;

$$Y_k = (X_k H_k)\left\{\frac{\sin \pi\varepsilon}{N sin\left(\frac{\pi\varepsilon}{N}\right)}\right\}. e^{j\pi\varepsilon(N-1)/N} + I_k + W_k. \tag{3.3.7}$$

The first component is the modulation value $X_k$ modified by the channel transfer function. This component experiences an amplitude reduction and phase shift due to the frequency offset. Since $N$ is always much greater than $\pi\varepsilon$, $N sin(\pi\varepsilon/N)$may be replaced by $\pi\varepsilon$.

The second term is the ICI caused by the frequency offset and is given by

$$I_k = \sum_{\substack{l=-K \\ l\neq k}}^{K} (X_k H_k)\left\{\frac{\sin \pi\varepsilon}{N sin\left(\frac{\pi(l-k+\varepsilon)}{N}\right)}\right\}. e^{j\pi\varepsilon(N-1)/N} e^{-j\pi(l-k)/N} \tag{3.3.8}$$

*Frequency Offset Estimation:*

If an OFDM transmission symbol is repeated, one receives, in the absence of noise, the *2N* point sequence

$$r_n = \left(\frac{1}{N}\right)\left[\sum_{k=-K}^{K} X_k H_k e^{\frac{2\pi jn(k+\varepsilon)}{N}}\right]; \quad n = 0,1,2,\ldots\ldots.2N-1. \tag{3.3.9}$$

The $k^{th}$ element of the *N* point DFT of the first *N* point of *eq.3.3.9* is

$$R_{1k} = \sum_{n=0}^{N-1} r_n e^{-2\pi jnk/N}; \qquad k = 0,1,2, \dots \dots \dots N-1, \qquad (3.3.10)$$

And the $k^{th}$ element of the DFT of the second half of the sequence is

$$R_{2k} = \sum_{n=N}^{2N-1} r_n e^{-\frac{2\pi jnk}{N}}$$

$$= \sum_{n=0}^{N-1} r_{n+N} e^{-\frac{2\pi jnk}{N}}; \quad k = 0,1,2.\dots..N-1 \qquad (3.3.11)$$

But from *eq.3.3.9*,

$$r_{n+N} = r_n e^{2\pi j\varepsilon} \rightarrow R_{2k} = R_{1k} e^{2\pi j\varepsilon} \qquad (3.3.12)$$

Including the AWGN one obtains

$$\begin{matrix} Y_{1k} = R_{1k} + W_{1k} \\ Y_{2k} = R_{1K} e^{2\pi j\varepsilon} + W_{2K} \end{matrix}; \qquad k = 0,1,2, \dots \dots \dots..N-1. \qquad (3.3.13)$$

Observe that between the very first and second DFT's, the ICI and the signal are altered in a similar way, by a phase shift proportional to frequency offset. Therefore, if offset is estimated using observations eq.3.3.13 it is possible to obtain accurate estimates even when the offset is too big for satisfactory data demodulation.

It is shown in [18] that the maximum likelihood estimate (MLE) of $\varepsilon$ is given by

$$\hat{\varepsilon} = \left(\frac{1}{2\pi}\right) tan^{-1} \left\{ \frac{\left(\sum_{k=-K}^{K} Im[Y_{2k} Y_{1k}{}^{*}]\right)}{\left(\sum_{k=-K}^{K} Re[Y_{2k} Y_{1k}{}^{*}]\right)} \right\}. \qquad (3.3.14)$$

## 3.4 BLIND CFO ESTIMATION USING CYCLIC PREFIX

Consider two uncertainties in the receiver of the OFDM symbol: the uncertainty in the arrival time of the OFDM symbol (such ambiguity gives rise to a rotation of the knowledge symbols) and the uncertainty in carrier frequency (a difference in the area oscillators in the transmitter and receiver gives rise to a shift of all of the subcarriers). The initial uncertainty is modeled as a delay in the $\delta(k-\theta)$,where $\theta$ could be the integer-valued unknown arrival time of a symbol. The latter is modeled as a complicated multiplicative distortion of the received data in the time domain $e^{j2\pi\varepsilon k/N}$,where $\varepsilon$ denotes the difference in the transmitter and receiver oscillators as a portion of the intercarrier spacing Notice that all subcarriers experience exactly the same shift $\varepsilon$. Considering $\varepsilon$as a uncertainty and the AWGN thus yield the received signal [19]

$$r(k) = s(k)e^{\frac{j2\pi\varepsilon k}{N}} + n(k). \tag{3.4.1}$$

Now take into account the transmitted signal s(k).This is obviously the DFT of the info symbols x(k) , which we assume are independent. Hence, s(k) is really a linear combination of independent, identically distributed random variables. Without frequency offset, the frequency response of each sub channel is zero at all of the subcarrier frequencies, i.e. the sub channel doesn't restrict one other. The aftereffect of a volume offset is a lack of orthogonality concerning the tones. The effective signal-to-noise ratio (SNRe) as a result of both additive noise and ICI is demonstrated to be lower bounded by

$$SNR_e(\varepsilon) \geq \frac{SNR}{1 + 0.5947SNR\,sin^2\pi\varepsilon}\left(\frac{sin\pi\varepsilon}{\pi\varepsilon}\right)^2 \tag{3.4.2}$$

Where,

$$SNR = \frac{\sigma^2{}_s}{\sigma^2{}_n},\quad \sigma_s{}^2 \triangleq E\{|s(k)|^2\}\; and\; \sigma^2{}_n \triangleq E\{|n(k)|^2\}.$$

The difference between the *SNR* and the *SNR<sub>e</sub>* is a measure of the sensitivity to a frequency offset $\varepsilon$.

*ML Estimation:*

Assume that folks observe 2N+L consecutive types of r(k), and these samples contain one complete (N+L)-sample OFDM symbol. The career with this symbol within the observed block of samples, however, is unknown as the channel delay is unknown to the receiver. Define the index sets

$$I \triangleq \{\theta, \ldots\ldots\ldots \theta + L - 1\}\; and$$
$$I' \triangleq \{\theta + N, \ldots\ldots\ldots \theta + N + L - 1\}$$

The set I' thus provides the indices of the info samples that are copies in to the cyclic prefix, and the set I provides the indices of the prefix. Collect the observed samples in the *(2N+L)* *1-vector $\boldsymbol{r} \triangleq [r(1) \dots \dots \dots . . r(2N+L)]^{\boldsymbol{T}}$.Notice that the samples in the cyclic prefix and their copies *r(k)* , $k \in IUI'$ are pairwise correlated, i.e.,

$$\forall k \in I: E\{r(k)r^*(k+m)\} = \begin{cases} \sigma^2{}_s + \sigma^2{}_n & m = 0 \\ \sigma^2{}_s e^{-j2\pi\varepsilon} & m = N \\ 0 & otherwise \end{cases} \tag{3.4.3}$$

While the remaining samples *r(k),k∉ IUI’* are mutually uncorrelated.

The log-likelihood function for $\theta$ and $\varepsilon$, $\Lambda(\theta, \varepsilon)$ is the logarithm of the probability density function $\hat{f}(\boldsymbol{r}|\theta,\varepsilon)$ of the *2N+L* observed samples in ***r*** given the arrival time $\theta$ and the carrier frequency offset $\varepsilon$. Using the correlation properties of the observations ***r,*** the loglikelihood function can be written as [19]

$$\begin{aligned} \Lambda(\theta, \varepsilon) &= log f(\boldsymbol{r}|\theta,\varepsilon) \\ &= log\left(\prod_{k\in I} \frac{f(r(k), r(k+N))}{f(r(k)r(k+N))} \prod_{k} f(r(k))\right) \end{aligned} \tag{3.4.4}$$

Since the ML Estimation of θ and ε is the argument maximizing$\Lambda(\theta, \varepsilon)$, we may omit this factor.

Under the assumption that***r*** is a jointly Gaussian vector, *eq.3.4.4* is shown in [19] to be

$$\Lambda(\theta, \varepsilon) = |\gamma(\theta)| \cos\big(2\pi\varepsilon + \angle\gamma(\theta)\big) - \rho\Phi(\theta) \tag{3.4.5}$$

Where ∠ denotes the argument of a complex number

$$\gamma(m) \triangleq \sum_{k=m}^{m+L-1} r(k)r^*(k+m), \tag{3.4.6}$$

$$\Phi(m) \triangleq \sum_{k=m}^{m+L-1} |r(k)|^2 + |r(k+N)|^2 \tag{3.4.7}$$

and

$$\rho \triangleq \left| \frac{E\{r(k)r^*(k+N)\}}{\sqrt{E\{|r(k)|^2\}E\{|r(k+N)|^2\}}} \right|$$

$$= \frac{\sigma^2{}_s}{\sigma^2{}_s + \sigma^2{}_n} = \frac{SNR}{SNR+1} \tag{3.4.8}$$

could be the magnitude of the correlation coefficient between r(k) and r(k+N). The first term in eq.3.4.5 could be the weighted magnitude that is a quantity of L consecutive correlations between pairs of samples spaced N samples apart. The maximization of the log-likelihood function may be performed in two steps:

$$\max_{(\theta,\varepsilon)} \wedge(\theta,\varepsilon) = \max_{\theta}\max_{\varepsilon} \wedge(\theta,\varepsilon) = \max_{\theta} \wedge\big(\theta, \hat{\varepsilon}_{ML}(\theta)\big).$$

The maximum with respect to the frequency offset $\varepsilon$ is obtained when the cosine term in *eq.3.4.5* equals one. This yields the ML estimation [19] of $\varepsilon$

$$\hat{\varepsilon}_{ML} = -\frac{1}{2\pi}\angle\gamma(\theta) + n \tag{3.4.9}$$

Where n is an integer.

# Chapter 4

# MODIFIED ML CFO ESTIMATOR

## 4.1 MODIFIED BLIND ML CFO ESTIMATION USING REPETITION OF DATA SYMBOLS

Carrier frequency offset (CFO) estimators based on the CFO-induced phase rotation have been discussed in [18] and [19]. The approach discussed in [19] is bandwidth efficient while there's no extra overhead is necessary and simply take advantage of the redundancy of the cyclic prefix (CP). Numerous techniques [30]-[31] exploit a preamble with a repetitive frame in time domain . On the inspiration of the orthogonality among the information-carrying subcarriers and the guard bands, subspace-based estimators have now been developed. One CFO estimator [31] exploits a preamble that's consists of pilot tones. The fundamental notion of the approach is to estimate the CFO by matching the spectral choice of the received preamble with the first pattern of the pilot tones.

Albeit the estimation of CFO using ML estimation approaches produce better performance than some algorithms discussed above and can meet close to the metaphysical Cramer-Rao lower bound (CRLB) on the mean square error, their complexity is usually regarded essentially the absolute most number of higher. However, approximations on the ML approaches offer satisfying suboptimum algorithms.

Motivated by these discussions, ML estimation (considering dispersive channels) having low complexity with a complexity (~O(N)) that's identical compared to that of the elementary approach discusssed in [18] (considering the AWGN channels) is considered here, where $N$ is the total number of subcarriers. The channel length is apparently exploited in symbol synchronization to boost the performance. Furthermore, this technique exploits a approach that uses channel length to boost the CFO estimation. Channel length is assumed to be known at the receiver.

In this technique CFO is estimated for OFDM systems, on the basis of the redundancy of the repeated data symbols is presented. The main element is unquestionably to work very well with fact a particular structure to the received signals is introduce by the repeated data symbols. Thus, by using the distinctive correlation characteristics of the repeated data symbols at each sampling time over dispersive channels the likelihood function is formulated. We're let's genuinely believe channel length is known at the receiver.

*Orthogonal Frequency Division Multiplexing Signal Model and Correlation Characteristics:*

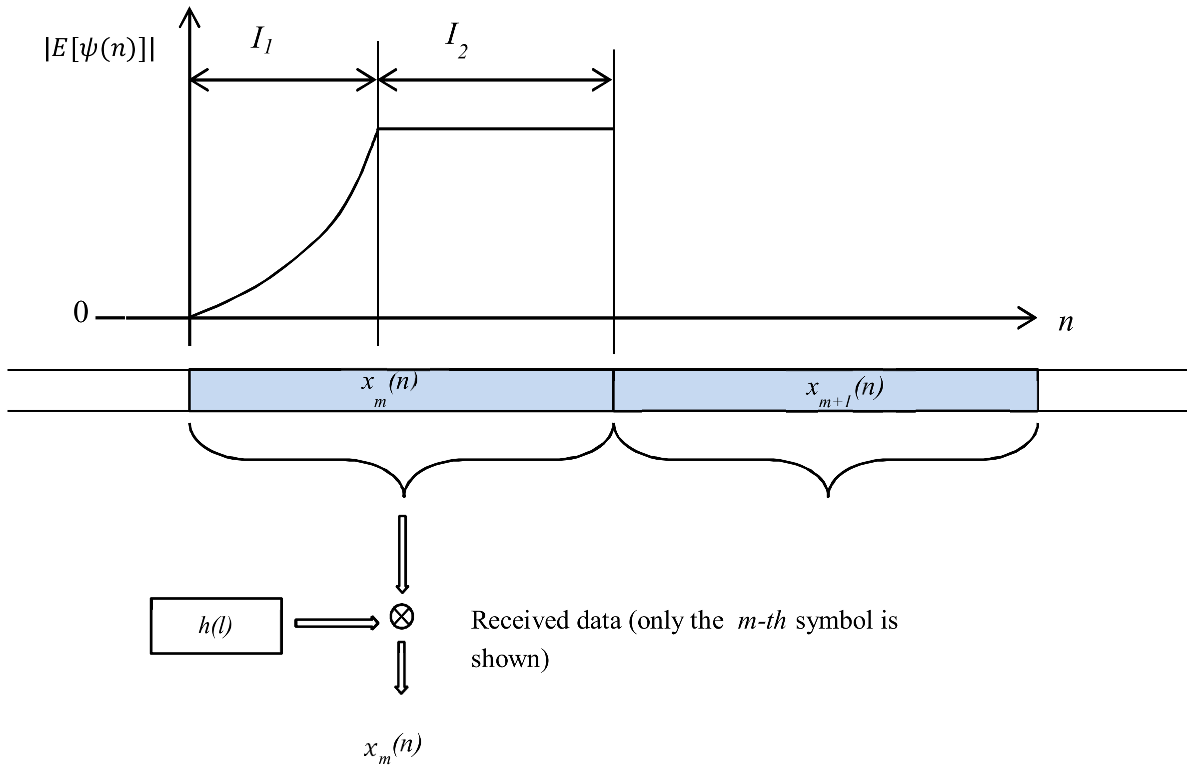


*Figure 4.1.1 Signal model that generated the correlation Characteristics, where $\otimes$ is the linear convolution.*

.

Channel taps are assumed to be quasi stationary on the 2 consecutive symbols. Consider an OFDM with $N$ subcarriers. The complex data are modulated onto the $N$ subcarriers in the shape of the inverse discrete Fourier transform (IDFT). Initially of every and every OFDM symbol the cyclic prefix (CP) might be inserted to prevent ISI and preserve the mutual orthogonality of the subcarriers. However, since we're repeating symbols so we're not inserting CP to be able to avoid further decline the bandwidth efficiency.

Following parallel-to-serial conversion, the current $m_{th}$ OFDM symbol $x_m(n), n \in \{0,1,2 \ldots \ldots . N-1\}$, is finally transmitted through a time-dispersive channel $h(l), l \in 0,1,2 \ldots \ldots L$. For $n \notin \{0,1, \ldots \ldots \ldots . N-1\}$, $x_m(n)$ is zero. Owing to the repeated data symbols transmitted data have the following characteristics: For $\forall n, if\ n_2 \neq n_1$, then the correlation of $x_m(n)$ at $n_1$ and $x_{m+1}(n)$ at $n_2$ $is\ E[x_m(n_1)x_{m+1}(n_2)] = 0$; Otherwise, $E[x_m(n_1)x_{m+1}(n_2)] = \sigma^2{}_x$, where $\sigma^2{}_x$ is the signal power.

As $x_m(n)$ and $x_{m+1}(n)$ are correlated, the correlation between $x_m(n)$ and $x_{m+1}(n)$ can

be expressed as

$$E[\psi_m(n)] = \begin{cases} \sigma^2{}_x e^{-j2\pi\varepsilon} \sum_{l=0}^{n} |h(l)|^2, & n \in I_1 \\ \sigma^2{}_x e^{-j2\pi\varepsilon} \sum_{l=0}^{L} |h(l)|^2, & n \in I_2 \end{cases} \tag{4.1.1}$$

Where

$$\psi_m(n) = \tilde{x}_m(n)\tilde{x}^*{}_{m+1}(n) \tag{4.1.2}$$

$I_1 \equiv \{0,1,2, \ldots \ldots \ldots L-1\}, I_2 \equiv \{L, L+1, L+2, \ldots \ldots \ldots N-1\}$. As a result of Scattering of the *h(l )* is uncorrelated and it can also be represented as

$$E[\tilde{x}_m(n)\tilde{x}^*{}_{m+1}(n+n_1)] = 0, \qquad for\ n_1 \neq 0. \tag{4.1.3}$$

*Fig.4.1.1* plots the magnitude of $E[\psi_m(n)]$. Its shape depends upon the particular channel condition.

*Modified Maximum likelihood Estimation:*

*A.LL Function:*

The model of the received data $\tilde{x}_m(n)$ after sampling can approximately d as a sophisticated Gaussian random variable in line with the central limit theorem, when N is large ; the probability density function (pdf) is given by [32]

$$f(\tilde{x}_m(n)) = \frac{\exp(-\frac{|\tilde{x}_m(n)|^2}{\sigma^2{}_{\tilde{x}} + \sigma^2{}_w})}{\pi(\sigma^2{}_{\tilde{x}} + \sigma^2{}_w)} \qquad \forall n$$

Where

$$E[|\tilde{x}_m(n)|^2] = \sigma^2{}_x \sum_l |h(l)| + \sigma^2{}_w \equiv \sigma^2{}_{\tilde{x}} + \sigma^2{}_w \tag{4.1.4}$$

With *eq.4.1.1* and *eq.4.1.3*, owing to the repeated data symbols, the received samples $\tilde{x}_m(n)$ and $\tilde{x}_{m+1}(n)$ are jointly Gaussian [32] with pdf

$$f\big(\tilde{x}_m(n), \tilde{x}_{m+1}(n)\big) = \frac{\exp\left(\dfrac{-|\tilde{x}_m(n)|^2 - 2\rho_n Re\{e^{j2\pi\varepsilon}\tilde{x}_m(n)\tilde{x}^*{}_{m+1}(n)\} + |\tilde{x}_{m+1}(n)|^2}{(\sigma^2{}_{\tilde{x}} + \sigma^2{}_w)(1-\rho^2{}_n)}\right)}{\pi^2(\sigma^2{}_{\tilde{x}} + \sigma^2{}_w)^2(1-\rho^2{}_n)} \tag{4.1.5}$$

Where

$$\rho_n \equiv \left|\frac{E[\tilde{x}_m(n)\tilde{x}^*{}_{m+1}(n)]}{\sqrt{E[|\tilde{x}_m(n)|^2]E[|\tilde{x}_{m+1}(n)|^2]}}\right| \tag{4.1.6}$$

Given an observation window of length *2N*, the likelihood function of the received samples $\tilde{\boldsymbol{x}}_m = [\tilde{x}_m(1), \tilde{x}_m(2), \dots\dots\dots.\tilde{x}_m(2N)]^T$,where $(.)^T$ denotes the transpose can be written as

$$\begin{aligned}\Lambda_m(\varepsilon) &= f(\tilde{\boldsymbol{x}}_m|\varepsilon)\\ &= \prod_{n\in I} f(\tilde{x}_m(n), \tilde{x}_{m+1}(n)) \prod_{n\notin IUI\prime} f\big(\tilde{x}_m(n)\big)\\ &= \prod_{n\in I} \frac{f(\tilde{x}_m(n), \tilde{x}_{m+1}(n))}{f(\tilde{x}_m(n))f(\tilde{x}_{m+1}(n))} \prod_n f(\tilde{x}_m(n))\end{aligned} \tag{4.1.7}$$

Where

$I \equiv I_1 U I_2$and *I'*≡the interval after shifting *I* to the right by *N* samples.

The term $\prod_n f(\tilde{x}_m(n))$ is independent of relative frequency offset $\varepsilon$ and can be neglected. Hence, the LL function can be written as**,**

$$\Lambda_m(\varepsilon) = \prod_{n\in I} \frac{f(\tilde{x}_m(n), \tilde{x}_{m+1}(n))}{f(\tilde{x}_m(n))f(\tilde{x}_{m+1}(n))} \tag{4.1.8}$$

The log-likelihood (LL) function [32] after receiving the *M* symbols, can be consideredas

$$\Lambda(\varepsilon) = \log\left(\prod_m \Lambda_m(\varepsilon)\right) \tag{4.1.9}$$

Combining *eq.4.1.4*, *eq.4.1.5* and *eq.4.1.8* and inserting them into *eq.4.1.9* yields the LL function as [32]

$$\Lambda_n = \frac{2(\rho_n\Gamma(n) - {\rho^2}_n\Phi(n))}{\overline{\Phi(n)}(1-{\rho^2}_n)} - \log\left(1-{\rho^2}_n\right)$$

$$= \frac{2\left(\rho_n|\Psi(\mathrm{n})|\cos\left(2\pi\varepsilon + \angle\Psi(\mathrm{n})\right) - {\rho^2}_n\Phi(n)\right)}{\overline{\Phi(n)}\left(1-{\rho^2}_n\right)} - \log\left(1-{\rho^2}_n\right) \qquad \ldots(4.1.10)$$

Where

$$\Gamma(n) \equiv \frac{1}{M}\sum_m \gamma\,(n)$$

$$\Phi(n) \equiv \frac{1}{M}\sum_m \phi\,(n)$$

$$\gamma_m(n) \equiv Re\left\{e^{j2\pi\varepsilon}\psi(n)\right\}$$

$$\phi_m(n) \equiv \frac{1}{2}(|\tilde{x}_m(n)|^2 + |\tilde{x}_{m+1}(n)|^2)$$

$$\Psi(n) \equiv \frac{1}{M}\sum_m \psi_m(n). \qquad \ldots(4.1.11)$$

Notice that

$$\Gamma(n) = \frac{1}{M}\sum_m \gamma\,(n) = Re\left\{e^{j2\pi\varepsilon}\frac{1}{M}\sum_m \psi_m\,(n)\right\} = Re\left\{e^{j2\pi\varepsilon}\Psi(n)\right\}$$

$$= |\Psi(n)|\cos\left(2\pi\varepsilon + \angle\Psi(n)\right). \qquad \ldots(4.1.12)$$

*CFO Estimation:*

The maximum LL function [see (4.1.10)], w,r.t relative frequency offset ε is obtained when cosine term equal to 1. To improve the CFO estimation accuracy, $\hat{\varepsilon}$ is averaged over all samples in Ias

$$\hat{\varepsilon} = -\frac{1}{2\pi}\angle\sum\nolimits_{n\in I}\Psi(n) \qquad (4.1.13)$$

Before performing the angle operation the common is simply taken. Only one angle operation is needed for the estimation of CFO using *eq.4.1.13*.Here the estimation of CFO different from that presented in [18] in the sense of kind of averaging taken and the angle operation is performed after averaging, i.e. we have improved performance and less computational complexity. And also in [18] estimation is performed in frequency domain whereas in proposed technique estimation is performed in time-domain i.e. before DFT operation.

## 4.2 SIMULATION RESULTS AND DISCUSSION

Extensive computer simulations are done to validate our proposed Modified ML method. Monte Carlo simulations are conducted to evaluate the performance of the modified estimator. The performance of modified method is compared with three methods; first one is cyclic prefix based CFO estimation [19] which is denoted by CPB estimation, second technique we used to compare can also be a cyclic prefix based CFO estimation [32] but this really is much the better compared to the technique given in [19]. Notation useful for this technique is Mod-CPB, i.e. modified cyclic prefix based technique. Third technique useful for comparison is CFO estimation centered on symbol repetition [18] and notation useful for this technique is SR, i.e. symbol repetition method. Our modified technique can also be centered on symbol repetition ML estimation technique but different from the technique given in [18]. Notation useful for Modified ML technique is Prop-SR. The estimator performance is evaluated by using normalized mean square error (MSE) and is given by

$$MSE = \frac{1}{P}\sum_{p=1}^{P}\frac{|\hat{\varepsilon} - \varepsilon|^2}{(2\pi/N)^2}$$

Where

P is the number of Monte Carlo runs and $\hat{\varepsilon}$ is the estimated CFO.

*Fig. 4.2.1* and *fig.4.2.2* plots the normalized CFO MSE of the modified method as a function of SNR in dB over the dispersive channel. Channel is assumed to be constant over entire observation window i.e for 20 OFDM symbols. Following are the parameters used in simulation:

- Number of Subcarriers = 64
- Normalized Frequency Offset = 0.01

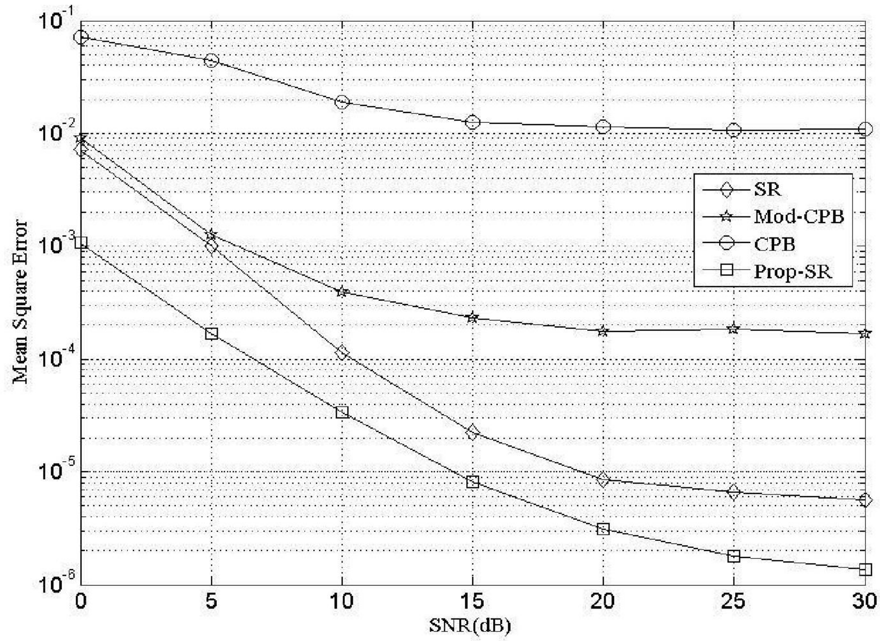


*Figure 4.2.1 MSEs of the CFO of the proposed estimator, under M=20 as a function of SNR over dispersive channel 1*

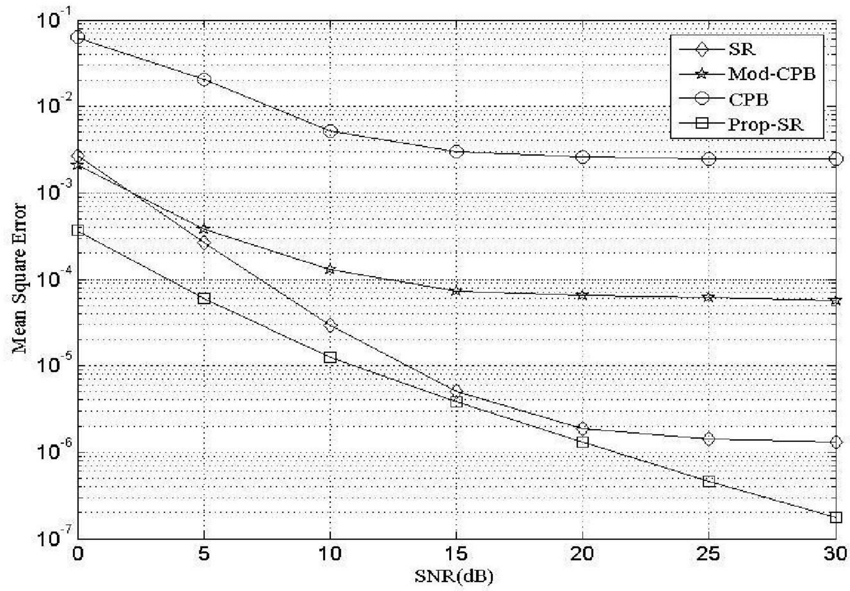


*Figure 4.2.2 MSEs of the CFO of the proposed estimator, under M=20 as a function of SNR over dispersive channel 2*

- Number of symbols used for estimation = 20
- Number of Monte Carlo runs = 1000
- Dispersive channel 1 with channel-tap powers = [0.0849 0.0011 0.1670 0.0402 0.0070

0.0112 0.2333 0.0000 0.1079 0.0005 0.1294 0.0004 0.0399 0.1484 0.0027 0.0001 0.0261]

- Dispersive channel 2 with channel-tap powers = [0.1462 0.0045 0.1815 0.15530 0.0125 0.4142 0.0789 0.0000 0.0070];

Fig.4.2.1 and fig.4.2.2 suggests that modified SR technique Prop-SR is optimum. The performance of Mod-CPB diverges from that of SR at SNR = 5dB. When the channel is dispersive, the performance of the CPB becomes much worse compared to Prop-SR method. As channel length decreases performance of Prop-SR method becomes better. Prop-SR technique gives around 4dB better performance than SR technique. The reason being modified SR technique performs estimation in time-domain whereas SR technique performs estimation in frequency domain i.e. after DFT operation. And also as angle operation is performed after averaging in proposed technique, the disturbances due to variation in correlation can be averaged out.

CFO estimation range of SR technique is limited to 25% of the subcarriers spacing, whereas Prop-SR technique is able to estimate the CFO of around 50% of subcarriers spacing. *Fig.4.2.3* shows the performance of SR and Prop-SR technique for CFO 25% and 45% of the subcarrier spacing. Figure shows that at 0.45 CFO the performance of SR method becomes worse, while the performance of Prop-SR method is approximately same for 0.25 and 0.45 CFO. Number of subcarriers used are 64 and channel is taken as dispersive channel.

In *fig.4.2.1*, *fig.4.2.2*, and *fig.4.2.3* we have assumed that Carrier Frequency Offset is constant over all OFDM symbols. *Fig.4.2.4* shows the performance of methods considering CFO as a uniformly distributed random variable lies in the range [-0.4,0.4], i.e. lies within 40% of the subcarrier spacing. *Fig.4.2.4* shows that Prop-SR technique performs best in this condition, whereas SR technique shows much more variations in its performance. This is because SR technique performs well only when CFO lies within 30% of subcarrier spacing, whereas Prop-SR method gives good performance up to 50 % of subcarrier spacing. This is due to the averaging which we have applied in Prop-SR technique which nullifies the effects of variations in correlation. *Fig.4.2.4* also shows that Prop-SR technique also outperforms the other two CPB and Mod-CPB techniques.

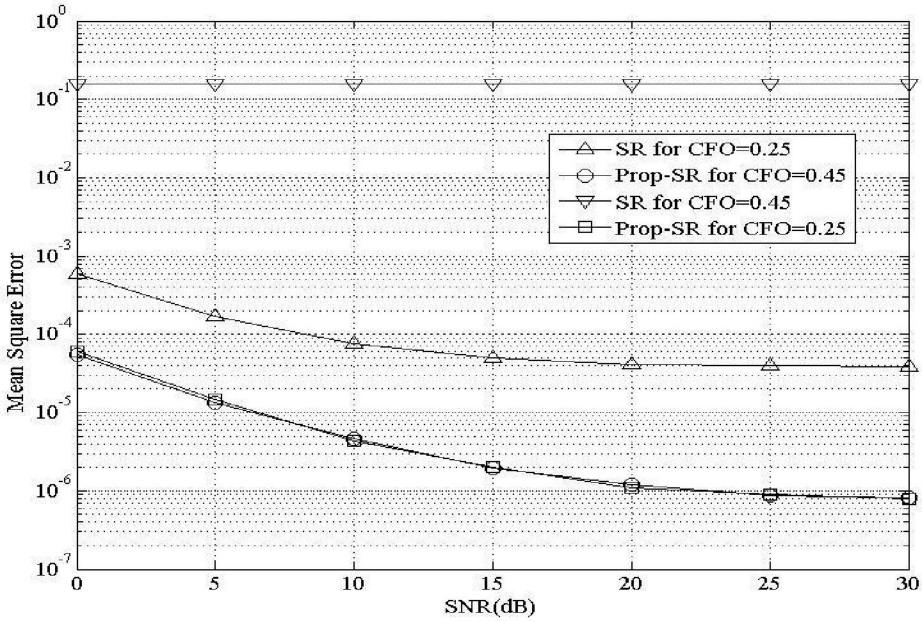


*Figure 4.2.3 MSEs of the CFO of the modified estimator, for CFO=0.25 and 0.45.*

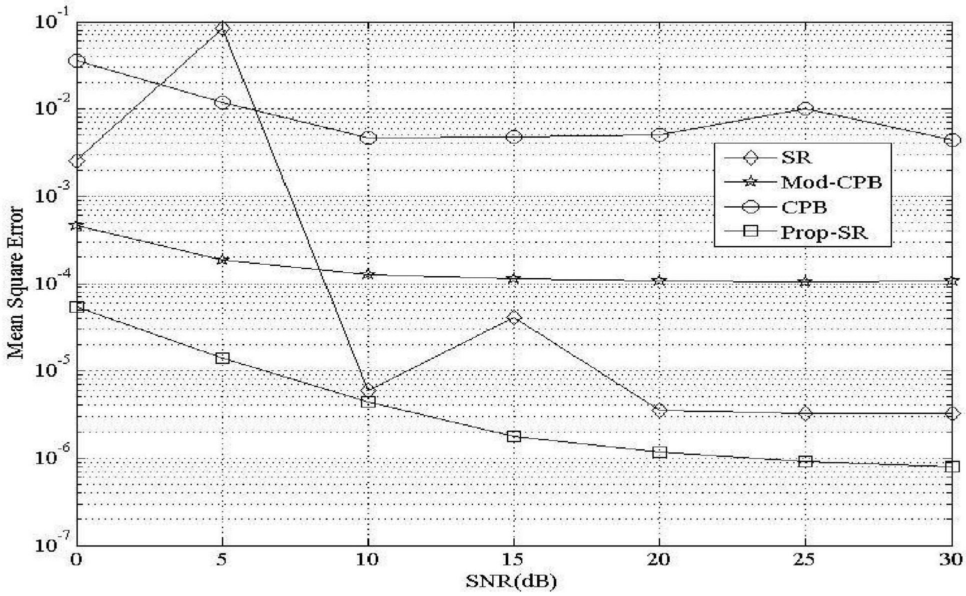


*Figure 4.2.4 MSEs of the CFO of the modified estimator, for uniformly distributed CFO under M=20 over dispersive fading channel.*

*Fig.4.2.5* shows the performance of Prop-SR technique in dispersive fading channel with low Doppler shift, i.e. channel changes for every OFDM symbol. Notice that the performance of SR method over slow-fading channel becomes poor than CPB and Mod-CPB after 10 dB, whereas Prop-SR technique outperforms all the techniques.

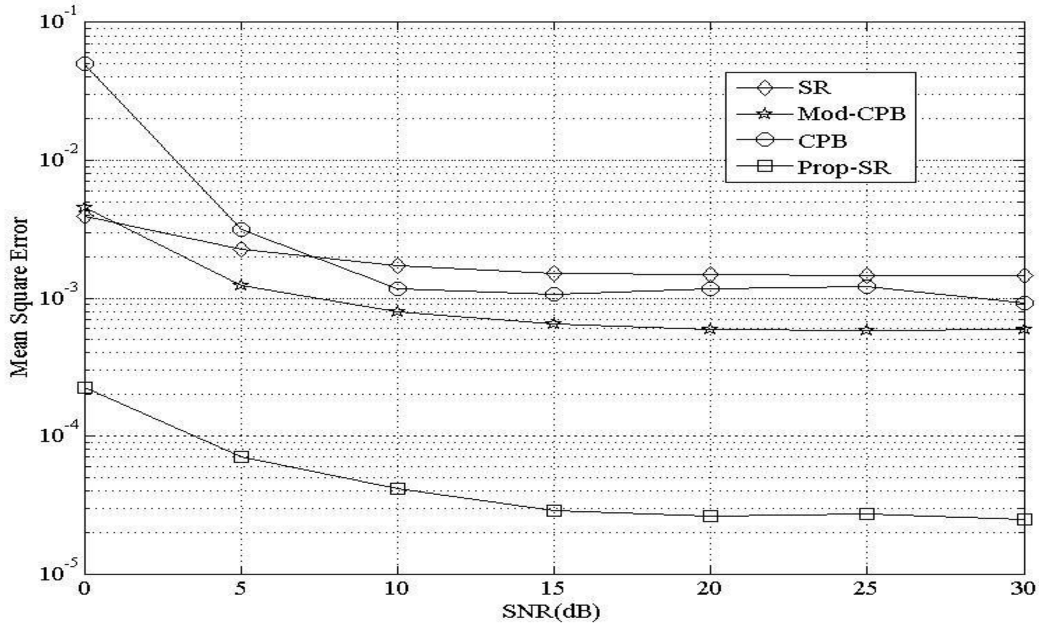


*Figure 4.2.5 MSEs of the CFO of the modified estimator, under M=20 as a function of SNR over low Doppler shift channel.*

*Fig.4.2.6* shows the performance of all four techniques in AWGN channel, which shows that Prop-SR technique, outperforms among three techniques. It can be observed that Prop-SR technique gives approximately 10 dB better performance than SR technique.

*Fig.4.2.7* shows the performance of SR and Prop-SR technique for number of symbols M=20 and M=40 used for CFO estimation. Figure shows that there is not much difference in performance for SR technique; however Prop-SR technique shows approximately 2.5 dB improvement in performance, i.e. as we increase number of symbols for estimation MSE for CFO decreases significantly.

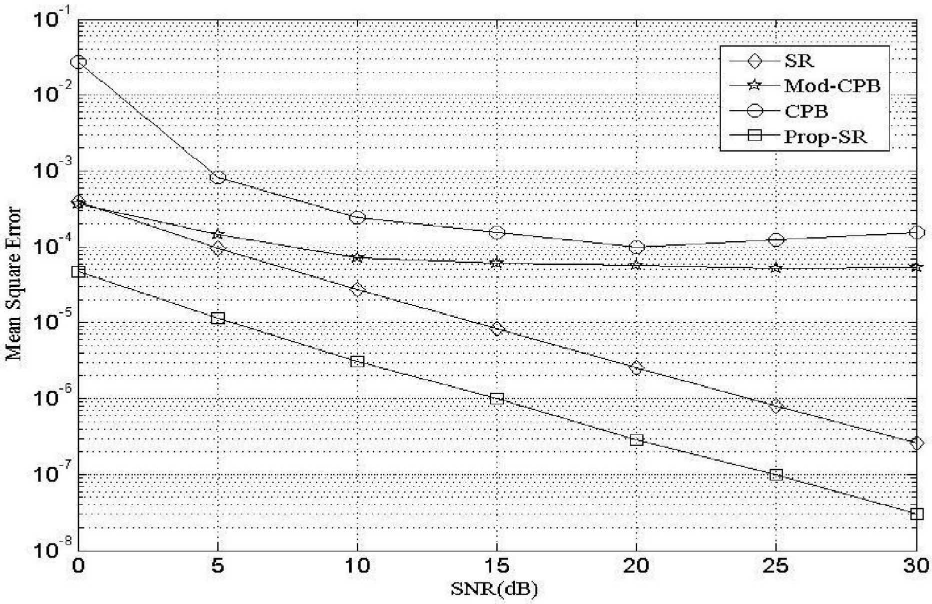


*Figure 4.2.6 MSEs of the CFO of the modified estimator, under M=20 as a function of SNR over AWGN channel.*

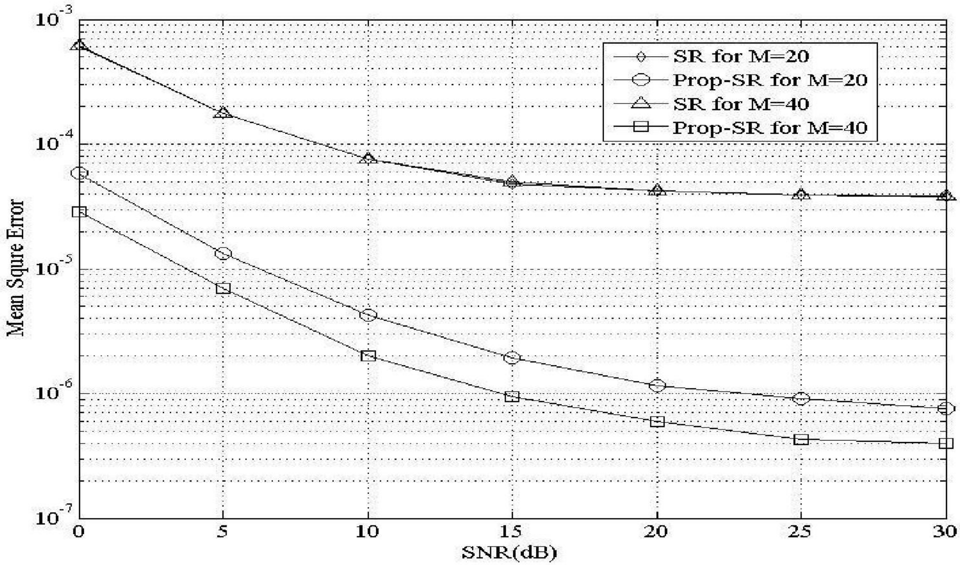


*Figure 4.2.7MSEs of the CFO of the modified estimator, under M=20 and M=40 as a function of SNR over Dispersive channel*

# Chapter 5

# CONCLUSION

OFDM is a great technique to handle impairments of the frequency selective channel. Hence, OFDM is a practical candidate for future 4G wireless communications techniques. On the other hand, one of the major drawbacks of the OFDM communication system is the drift in reference carrier. The offset present in received carrier will lose orthogonality among the carriers, and hence, the CFO causes a reduction of desired signal amplitude in the output decision variable and introduces ICI, then brings up an increase of BER. This leads to the necessity to estimate the CFO in order to cancel it in next stage. This thesis presents a modified ML Blind CFO estimator based on symbol repetition known as Prop-SR.The main advantage with the Prop-SR algorithm is that it gives much better performance in dispersive fading channel. However there is a cost to pay, which is reduced bandwidth efficiency due to data symbol repetition. But, symbol repetition is done only in the synchronization phase of symbol transmission (i.e. when CFO is estimated) so the performance of estimator can overcome the cost of reduced bandwidth efficiency. The Prop-SR algorithm is equipped with lower complexity and is computationally efficient with respect to its peer ones.

Prop-SR technique outperforms its similar SR technique in many conditions and it has been shown by simulation results that Prop-SR provides significant improvement in SNR over SR technique.

In terms of future work, it is worth to mention that this blind method may be applied in the OFDMA case and MIMO-OFDM. Many developments can be achieved by improving the estimation function.

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